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Discontinuous wave interaction with interfaces between anisotropic elastic media

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Abstract

The problem about dynamic interaction of discontinuous waves with interfaces between anisotropic elastic media is considered. To investigate this phenomenon accompanied by formation of reflected and refracted quasi-longitudinal and quasi-shear discontinuous waves, a technique based on joint usage of the zero approximation of the ray theory and method of stereomechanical impact is proposed. It is used for the analysis of the wave front transformation, scattering and focusing. The setup problem solutions can be applied to discovering the most seismically hazardous zones in the earth's crust, interpretation of geophysical data about geological rock structures and the analysis of the causes of dynamic delamination of layered composite and nanomaterials.

Keywords: Anisotropic media; Discontinuous waves; Interfaces, Interaction; Focusing; Scattering

1. Introduction

In studies of discontinuous (shock type) waves propagation in elastic media, the greatest attention is, as a rule, devoted to geometrical construction of the evolving field-function discontinuity surfaces and calculation of the discontinuity magnitudes, which provide the most complete information about the wave front transformation and the intensity of an impulse carried by the wave at each point of the front surface.

As to seismology these questions are topical for investigation of the wave processes taking place in the earth's crust and for description of behaviour of seismic waves in the vicinity of tectonic inhomogeneities, where the waves can endure the effects of focusing or scattering. These effects manifest themselves most

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clearly in the convex and concave parts of the interfaces between rock structures. It is known, that there are no ways of prognosticating and eliminating the earth-quakes, but using the questions solutions it is possible to find the tectonic regions, where the natural seismic waves can focus and concentrate their energy provoking collapse of above-ground and underground constructions or where the waves disperse without damage for the environment.

In the course of seismic reconnaissance of mineral resources the considered results are useful for theoretical interpretation of geophysical data about the explored geological rock structures.

The problem solution can also be used for the analysis of explosion wave influence on environment and for elaboration of rational methods for pursuance of explosion works eliminating the possibility of artificial earthquakes generation at predetermined regions.

The considered features are inherent also in mechanics of composites and nanomaterials. One of the most important advantages of the composite materials is their property to withstand shock loads and dissipate mechanical energy of vibrational motions. It is associated to some extent with non-homogeneity of the structure materials and existence of interfaces between the system subdomains, where the moving fronts of strain saltuses experience the reflection–refraction effects. In consequence of execution of several acts of this type effects the wave fronts diffract and the shock wave dissipates through transformation of the strain energy into heat. However these phenomena are very often accompanied by undesirable changes of the composite medium structure, correlated with disruption of bonds between its components and the structure delamination.

The discussed phenomena are usually characterized by short duration of highly intensive initial field of pressure, which at the initial stage of time is concentrated, as a rule, in a small domain adjacent to the zone of impact initiation of the wave and by transformation of the wave front surface as it propagates. Inasmuch as in this case the boundary of the domain chosen for calculation evolves with the wave front progress, the solution has to be found in the family of discontinuous functions evolving in time. So the traditional classical and numerical methods turn out to be of low efficiency for the analysis of the similar processes.

For the solution of the problems of such type a prominent role is played by the methods of geometrical optics (Fedorov, 1968; Karal and Keller, 1959; Ogilvy, 1990; Podilchuk and Rubtsov, 1988, 1996), used in the theory of field discontinuity front propagation in non-dispersive media. They are correlated with application of a ray coordinate system wherein aggregate of the coordinate surfaces coincides with the evolving surfaces of the non-stationary waves. Formally, this approach is realized through representation of the wave equation solution in terms of a ray series. By means of its use, an Eiconal equation and transport equations system are constructed. The former is a non-linear partial differential equation describing the front surface and the ray aggregate. So it is referred to as kinematic equation. The transport equations constitute a system of linear partial equations. They determine the field functions at the front surface and behind it. So these equations are referred to as dynamic ones. Obviously the field discontinuity generated at the front surface is described by the zeroth term of the ray series.

It should be emphasized that the equation system obtained in this way is also very complicated and the convergence of its solutions can be achieved in the simplest cases only. For this reason, usually the greatest attention is paid to the zeroth approximation of the ray method (Petrashen, 1980), providing good quantitative description of the wave phenomenon in a small vicinity of the wave front. Its application allows one to construct the evolving front, to determine the wave polarization vector at every point of its surface, to calculate the discontinuity magnitudes of the functions of strains and stresses and also the wave phase as functions of the ray coordinates. As to this method, the zeroth term of the ray series is taken into account, which can be calculated independently of other ones. This point has a simple mechanical explanation. It lies in the fact that every element of the discontinuous wave front moves with the sound speed. It has the largest value at the point considered and no other perturbation can move faster in its vicinity. So the dynamic perturbations from other points of the medium cannot overtake the particular element and affect on the value of its field function discontinuity.

In parallel with this, the zeroth approximation of the ray method used jointly with the locally plane approach (Petrashen, 1980; Gulyayev et al., 2002) allows one to state the problem about interaction of shock waves with interfaces between the elastic media possessing different mechanical properties. This problem is associated with the necessity to construct kinematically the front surfaces of reflected and refracted waves with different polarizations and to calculate dynamical parameters of the field discontinuities on these fronts.

The technique for solving the kinematic problem of the interactions like this is based on the analysis of the Snell equations. In the cases of transversally isotropic elastic media, these equations are essentially nonlinear. Gulyayev et al. (2000) proposed a procedure for their solution, which rests on combined application of the method of continuation by a parameter and the Newton method. To analyse peculiarities of the field discontinuity transformations in interfaces between elastic media, usually the conditions of stresses continuity are used (Fedorov, 1968; Petrashen, 1980). This approach is rather complicated, because it is not convenient to express the field discontinuities through stresses, especially in the cases of anisotropic media. But when the zeroth approximation of the ray method is used, an alternative method of attack becomes more rational, which relates to the above-stated effect of discontinuities values independence on dynamic perturbations located behind the front.

It is based on the integral formulation of physical laws, which are completely equivalent to the differential statement of the problem for the continuous processes. According to the mentioned principles, both some integral correlations between functions and their lowest derivatives (the kinematic conditions of compatibility) as well as the law of momentum conservation are satisfied in the moving discontinuities surfaces. This idea (Gulyayev et al., 1997), based on the theorem about the momentum conservation at impact interaction (Gol'dsmit, 1960), was used for formulation of equations of dynamic interaction of a discontinuous wave with an interface between isotropic elastic media. In (Anikiev et al., 2000) this approach was used for theoretical and experimental investigation of the phenomenon of quasi-total internal reflection at the interface between water and elastic media. Here it is used in the framework of the ray method zeroth approximation for the description of the dynamical process at statement of the dynamic boundary conditions in the surfaces interfacing anisotropic media.

2. The problem statement

Let physical properties of a homogeneous anisotropic medium be defined by the elasticity parameters $c_{ik,pq} = \text{const.}$ and density $\rho = \text{const.}$ The medium motion is described by the equations

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq} \frac{\partial^2 u_q}{\partial x_k \partial x_p} - \frac{\partial^2 u_i}{\partial t^2} = 0 \quad (i = 1, 2, 3), \tag{1}$$

where x_1, x_2, x_3 are the Cartesian coordinates; u_1, u_2, u_3, the components of the elastic displacement vector; t the time; $\lambda_{ik,pq} = c_{ik,pq}/\rho$.

The system (1) solution is represented in the form of a plane monochromatic wave with wave number k and phase velocity v. Its fronts are surfaces of constant phase $\mathbf{n} \cdot \mathbf{r} - \mathrm{v} \cdot t = \text{const.}$, moving with the velocity $\mathbf{v} = \mathrm{v} \cdot \mathbf{n}$ and coinciding locally with areas perpendicular to the unit vector $\mathbf{n}$.

The wave polarization vector $\mathbf{A}$ and the phase velocity $\mathbf{v}$ are determined for the selected direction $\mathbf{n}$ on the basis of homogeneous algebraic equations relative to A_i (Fedorov, 1968; Petrashen, 1980)

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq} n_k n_p A_q - \mathrm{v}^2 A_i = 0 \quad (i = 1, 2, 3). \tag{2}$$

With the help of these equations one can find three values of the velocity

$$\mathrm{v}^{(1)}(\mathbf{n}) > \mathrm{v}^{(2)}(\mathbf{n}) \geqslant \mathrm{v}^{(3)}(\mathbf{n}) > 0$$

and polarization vectors $\mathbf{A}^{(r)}$ defining the displacement vectors

$$\mathbf{u}^{(r)} = u^{(r)}\mathbf{A}^{(r)} \quad (r = 1, 2, 3). \tag{3}$$

If a discontinuous wave is considered, the constitutive equations provide that its front surface may be represented by the correlation

$$\tau(x_1, x_2, x_3) - t = 0, \tag{4}$$

where the function $\tau(x_1, x_2, x_3)$ has to satisfy the first order partial differential equation (Petrashen, 1986)

$$\sum_{i,k,p,q=1}^{3} \lambda_{ik,pq} p_k p_p A_q^{(r)} A_i^{(r)} = 1.$$

It generalizes the Eikonal equation used in geometrical optics for the construction of systems of rays and fronts to the case of anisotropic elastic waves.

The quantities p_k $(k = 1, 2, 3)$ included in the above equation represent the components of the refraction vector $p_k \equiv \partial\tau/\partial x_k = n_k/\mathrm{v}^{(r)}(\mathbf{n})$ $(k = 1, 2, 3)$.

The above equation must be solved to be able to reconstruct the transforming fronts of the wave. Using the method of characteristics this equation is rearranged to the system of ordinary differential equations

$$\begin{aligned} &\frac{\mathrm{d}x_k}{\mathrm{d}\tau} = \xi_k = \sum_{i,p,q=1}^{3} \lambda_{ik,pq} p_p A_q^{(r)} A_i^{(r)}, \\ &\frac{\mathrm{d}p_k}{\mathrm{d}\tau} = 0 \quad (k = 1, 2, 3). \end{aligned} \tag{5}$$

The first group of these equations describes the wave propagation along the ray with the ray velocity $\xi = \xi^{(r)}(\mathbf{n}, x_k)$, which generally does not coincide with the phase velocity $\mathrm{v}^{(r)}$. In case of a homogeneous medium the ray is rectilinear, but it is not orthogonal to the phase front. Points on the surface of the front, at which the determinant of the matrix

$$\left\| \sum_{i,p,q=1}^{3} \lambda_{ik,pq} A_q^{(r)} A_i^{(r)} \right\| \quad (k = 1, 2, 3)$$

of the coefficients of the right-hand side of this system vanishes, are bifurcation points, since, in a small vicinity of them, two or more rays may correspond to one direction of the vector of the normal **n** (Arnold et al., 1984; Kravtsov and Orlov, 1980; Poston and Stewart, 1978; Gilmore, 1981). Hence, these points are located either on caustics or at focal points of the rays.

The system of rays and wave fronts constructed with the use of (5) enables one to proceed to determine the wave intensity in the neighbourhood of the wave front. For this purpose, it is convenient to expand the solution of (1) along the ray behind the front into the series

$$u_q = \sum_{m=0}^{\infty} u_q^{(m)}(x_1, x_2, x_3) f_m[t - \tau(x_1, x_2, x_3)] \quad (q = 1, 2, 3), \tag{6}$$

where the functions f_m, satisfying the correlations $f'_m(y) = f_{m-1}(y)$, are supposed to possess discontinuities of their derivatives, for example, of the order $n + 2$ (Petrashen, 1980).

In formulating the problem about the wave behaviour in a small vicinity behind its front and calculation of the stress discontinuity, only one term $m = 0$ is retained in (6), so

$$u_q = u_q^{(0)}(x_1, x_2, x_3) \cdot [t - \tau(x_1, x_2, x_3)] \tag{7}$$

and the vector $\mathbf{u}^{(0)}$ is calculated using the homogeneous system of equations

$$\sum_{k,p,q}^{3} \lambda_{ik,pq} p_k p_p u_q^{(0)} - u_i^{(0)} = 0 \quad (i = 1, 2, 3), \tag{8}$$

which have the solution (Petrashen, 1980)

$$u_q^{(0)} = \frac{c_0(\alpha, \beta) \cdot A_q^{(r)}(\alpha, \beta, \tau)}{\sqrt{J(\alpha, \beta, \tau)}} \quad (q = 1, 2, 3). \tag{9}$$

Here α, β, τ are the system of ray coordinates and the quantity

$$J = \frac{\partial(x_1, x_2, x_3)}{\partial(\alpha, \beta, \tau)} \tag{10}$$

is a functional determinant for conversion of the ray coordinates into Cartesian ones. It is the measure of the ray divergence in the ray tube.

The above relations make it possible to follow the evolution of a shock type wave front and calculate the field-function discontinuity on its surface, as it propagates in a homogeneous anisotropic medium.

3. Kinematics of a discontinuous wave interaction with a plane interface

In study of interaction of a discontinuous wave with an interface we are to solve two problems—kinematic and dynamic ones. The first problem is reduced to the construction of the front surfaces of the reflected and refracted waves produced at interaction of the incident wave with interface surface G between media I and II (Fig. 1). To set it up, a ''locally plane approach'' (Petrashen, 1980) is used, whereby all the front surfaces in the small vicinity of the point of the wave interaction with the interface (as also the

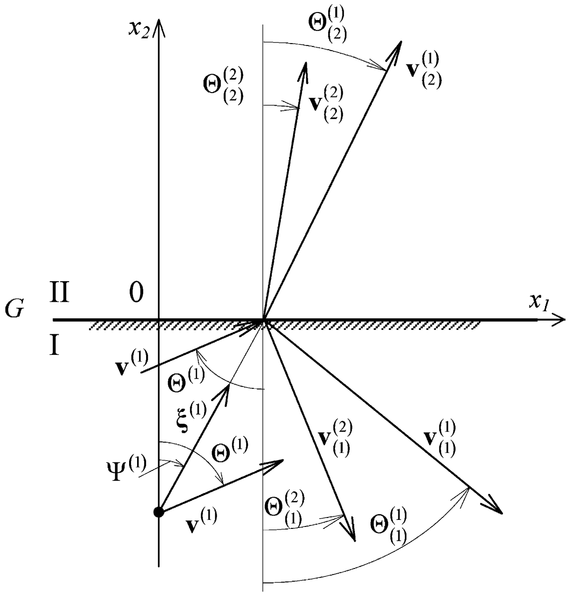

Fig. 1. Scheme of the wave velocities directions.

surface G is itself) are considered to be locally plane. It allows one to assume, that at reflection–refraction of a plane wave at the plane interface, reflected and refracted waves are generated, which also belong to the family of plane waves. The solution technique for the first problem, based on the use of the formulated premise, leads to the necessity to solve the generalized equations of Snell, which in case of transversally isotropic media take the form:

$$\frac{\sin \Theta^{(r)}}{\mathrm{v}^{(r)}(\Theta^{(r)})} = \frac{\sin \Theta_1^{(v)}}{\mathrm{v}_1^{(v)}(\Theta_1^{(v)})} = \frac{\sin \Theta_2^{(\mu)}}{\mathrm{v}_2^{(\mu)}(\Theta_2^{(\mu)})} \quad (r, v, \mu = 1, 2). \tag{11}$$

Here the designations are specified: the superscript in brackets denotes the wave with number $r, v, \mu = 1$ corresponding to quasi-longitudinal (quasi-primary) qP-wave and $r, v, \mu = 2$ corresponding to quasi-shear (quasi-secondary) qS-wave; the subscript in brackets indicates the medium number; the values $\mathrm{v}^{(r)}$ and $\Theta^{(r)}$ without subscripts relate to the incident wave propagation in the first medium.

The difference between correlations (11) and the usual form of the Snell law is in the denominators $\mathrm{v}^{(\mathrm{v})}(\Theta^{(\mathrm{v})}), \mathrm{v}^{(\mu)}(\Theta^{(\mu)})$ dependence on the unknown angles $\Theta^{(v)}$, $\Theta^{(\mu)}$ and, implicitly, on the incidence angle Θ. So, in order to find the values of angles $\Theta^{(v)}, \Theta^{(\mu)}$ $(v, \mu = 1, 2)$ corresponding to the preset Θ, it is necessary to solve the initial non-linear system. With this aim in view, the Newton method is used jointly with the method of continuation by a parameter (Gulyayev et al., 2000).

The condition of possible non-uniqueness of the system (11) solution corresponds to the convergence (tangency) and intersection of the reflected and refracted rays after the incident rays interaction with the interface G, while the aggregate of such critical states is related to the formation of an envelope of the family of rays, which is referred to as a caustic. The caustics give rise to the formation of geometrical singularities at the surfaces of the reflected and refracted wave fronts as a result of interaction of a regular incident wave front even with a plane boundary G.

Since the singularities of the wave fronts are generated on the caustics, they will also be focused on the caustics, giving rise to the vanishing of the functional determinant J in (9) and an unlimited increase in the field intensity at the points of geometrical singularities. At the caustics the wave phase is also reversed (Kravtsov and Orlov, 1980).

4. Dynamics of a discontinuous wave interaction with a plane interface

After the construction of the system of rays and fronts of the reflected and refracted discontinuous waves, it becomes possible to proceed to the investigation of dynamics of the incident wave interaction with the interface between two elastic media. With this aim in view, it is necessary to present the equations of boundary conditions at the interface surface

$$(\dot{\mathbf{u}}_{(1)} - \dot{\mathbf{u}}_{(2)})\big|_G = 0, \tag{12}$$

$$(\boldsymbol{\sigma}_{2(1)} + \boldsymbol{\sigma}_{2(2)})\big|_G = 0, \tag{13}$$

where $\dot{\mathbf{u}}_{(1)}$, $\dot{\mathbf{u}}_{(2)}$ are the velocity vectors of the media particles at the interface G; $\boldsymbol{\sigma}_{2(1)}$, $\boldsymbol{\sigma}_{2(2)}$, the vectors of density of the forces $\boldsymbol{\sigma}_2$ acting on the elementary area $x_2 = \text{const.}$ in media I and II, correspondingly.

But inasmuch as in our case the investigations are performed in the framework of the zeroth approximation of the ray method (Petrashen, 1980) and the problem is stated in terms of the velocity discontinuities of the elastic elements, it is more convenient to use the integral approach for the description of dynamical processes at interaction of the wave fronts and to resort to the methods of the theory of stereomechanical impact (Gol'dsmit, 1960). Furthermore, in analysis of the shock wave interactions with the curvilinear interface G the locally plane conception of the statement is applied.

To substantiate the validity of the used approach to the problem of a discontinuous wave interaction with an interface between two anisotropic elastic media it should be pointed out that in the framework of the zeroth approximation it is absolutely identical with the technique based on the application of equation (13) and in this paper it is used only for simplifying the algebraic and differential transformations.

At first, take into consideration a simple example. Let a short discontinuous impact propagate in a bar of unit area with discontinuous properties of elasticity module E_i and densityρ_i $(i = 1, 2)$ in its parts I and II separated by interface surface G (Fig. 2). Before impact interaction with G the particles in the discontinuous wave have velocity $\dot{u}^{\mathrm{I}}_{(1)}$, which is considered to be constant inside the wave with the length $\mathrm{v}_{(1)} \cdot \Delta t$. Here $\mathrm{v}_{(1)} = \sqrt{E_1/\rho_1}$ is the wave propagation velocity; Δt, the impact action duration; the indices I, R, T denote the incident, reflected and transmitted waves.

It can be written as

$$\sigma = E\varepsilon, \quad \varepsilon = \frac{\partial u}{\partial x}, \quad \dot{u} = \frac{\partial u}{\partial t}.$$

At the wave front the equalities are valid as follows:

$$x = \mathrm{v} \cdot t, \quad \mathrm{d}x = v \cdot \mathrm{d}t.$$

Herefrom it issues as

$$\varepsilon = \frac{\partial u}{\partial x} = \frac{\partial u}{\mathrm{v}\partial t} = \frac{1}{\mathrm{v}}\dot{u}.$$

Using these correlations, one gains

$$\sigma = E\varepsilon = \frac{E \cdot \dot{u}}{\mathrm{v}} = \rho\mathrm{v} \cdot \dot{u}$$

and

$$\sigma \cdot \Delta t = \rho\mathrm{v} \cdot \dot{u} \cdot \Delta t = \dot{u} \cdot \Delta m = \Delta Q.$$

Here $\Delta m = \rho\mathrm{v} \cdot \Delta t$ is the mass of the bar particles involved into the motion, ΔQ is the momentum of these particles.

Now consider the condition of momentum conservation for the bar elements involved into motion before the impact with the interface G and after the impact (Gol'dsmit, 1960)

$$\Delta\mathbf{Q}_{\mathrm{I}} = \Delta\mathbf{Q}_{\mathrm{T}} + \Delta\mathbf{Q}_{\mathrm{R}}. \tag{14}$$

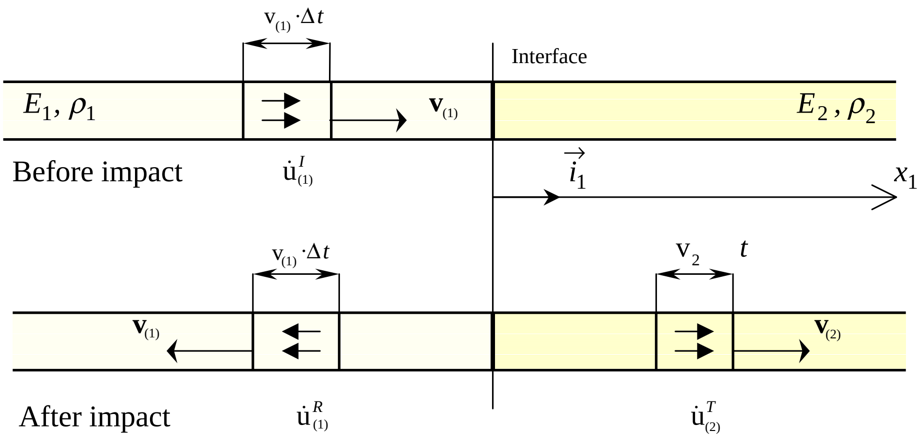

Fig. 2. Discontinuous wave interaction with the bar interface.

After appropriate substitutions, the equation that defines the connection between velocities of the particles of the incident, transmitted and reflected waves is deduced as follows (Fig. 2)

$$\rho_1 \mathrm{v}_{(1)} \dot{u}^{\mathrm{I}}_{(1)} = \rho_2 \mathrm{v}_{(2)} \dot{u}^{\mathrm{T}}_{(2)} - \rho_1 \mathrm{v}_{(1)} \dot{u}^{\mathrm{R}}_{(1)}. \tag{15}$$

This equation is identical to the equation

$$\sigma_{\mathrm{I}} = \sigma_{\mathrm{T}} - \sigma_{\mathrm{R}} \tag{16}$$

resulting from (13).

The identity shown suggests that the approach to the task of determination of the stress discontinuity interaction with the interface by the use of momentum conservation condition and stereomechanical theory of impact (Gol'dsmit, 1960) is equivalent to the technique based on application of the condition of the stress continuity in the form of Eq. (13).

To apply this technique to the considered problem for anisotropic elastic media initially stop on some preliminary remarks. Note, that a discontinuous wave front is surface (4) or

$$\frac{\mathbf{n} \cdot \mathbf{r}}{\mathrm{v}} - t = 0, \tag{17}$$

where $\mathbf{v} = \mathrm{v} \cdot \mathbf{n}$.

So

$$\mathrm{d}\frac{\mathbf{n} \cdot \mathbf{r}}{\mathrm{v}} = \mathrm{d}t. \tag{18}$$

Then it issues from (7) as

$$\dot{u}_q = \frac{\partial u_q}{\partial t} = u_q^{(0)}$$

and

$$\frac{\partial u_q}{\partial n} = \frac{\partial u_q}{\mathrm{v} \partial t} = \frac{1}{\mathrm{v}} \cdot u_q^{(0)}. \tag{19}$$

Equality (19) is valid at wave front surface (17) only and it testifies that the element strain $\partial u_q/\partial n$ and its velocity $\partial u_q/\partial t$ differ solely by the constant denominator v, for this reason they are equivalent at the problem statement.

It is important also to underline that the correlation

$$\dot{\mathbf{u}}^{(r)} = \dot{u}^{(r)} \mathbf{A}^{(r)} \tag{20}$$

follows from (3).

At statement of dynamic conditions of interaction of the incident, reflected and refracted discontinuous waves at interface, the zeroth approximation of the ray method is used. The locally plane statement of the problem in the small vicinity of the interface and the integral approach permit one to apply to the methods of the theory of stereomechanical impact (the principle of momentum conservation) for the phenomenon description.

According to this conception, now consider that a plane quasi-longitudinal shock type discontinuous qP-wave, propagating with phase velocity $\mathrm{v}^{(1)}$ in medium I, interacts at an incident angle $\Theta^{(1)}$ with a plane interface G, $x_2 = 0$ between media I and II. In the plane of symmetry $x_3 = 0$, which is perpendicular to the wave front and plane G, separate a quadrangle element of thickness $\mathrm{v}^{(1)} \cdot \Delta t$ by two rays l_1 and l_2 in medium I behind the front (Fig. 3). Here Δt is so small time segment that it becomes possible to neglect change of discontinuous components of the field functions behind the front and to consider them to be constant. Underline, that the last assumption is in correspondence with the zeroth approximation of the ray

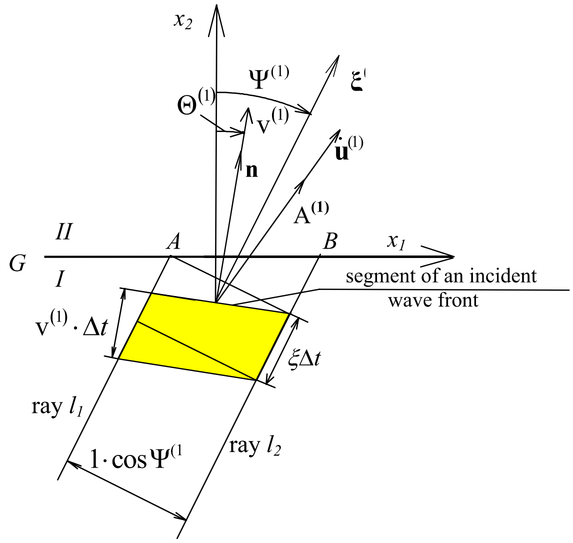

Fig. 3. Scheme of a discontinuous wave element motion.

method. Assume, that the rays l_1 and l_2 intersect G at the angle $\Psi^{(1)}$ and separate unit length segment AB in it. Then the distance between the rays measures $1 \cdot \cos \Psi^{(1)}$.

At interaction of the considered wave with the plane G, two plane reflected ($qP_{(1)}, qS_{(1)}$) and two plane refracted ($qP_{(2)}, qS_{(2)}$) waves, polarized also in the incident plane, are generated. As it takes place, two pairs of quadrangle elements between the appropriate rays behind the fronts of the generated waves correspond to the separated element behind the front of the incident wave (Fig. 4).

Reasoning from the integral approach in the theory of discontinuous waves, considering the conditions of kinematic compatibility and dynamic conservation in the wave front (Petrashen, 1980), the description of

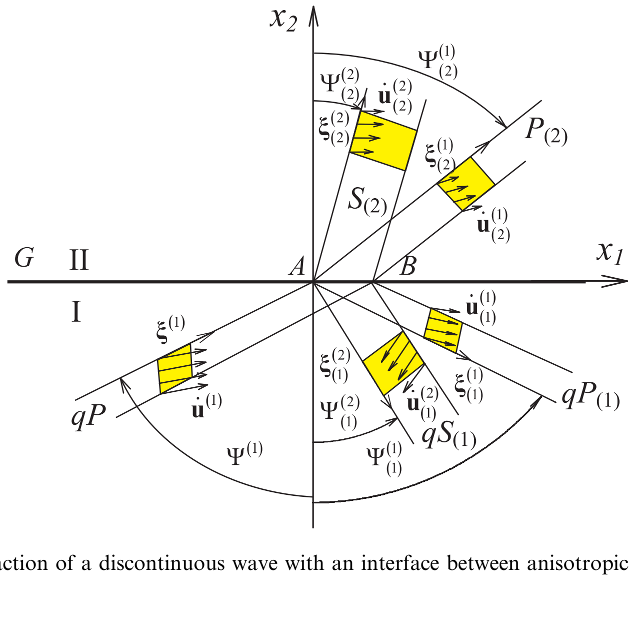

Fig. 4. Interaction of a discontinuous wave with an interface between anisotropic elastic media.

dynamic interaction of this waves in the interface G will be performed on the basis of the principle of momentum conservation at impact (Gol'dsmit, 1960) applied to the elements of the media I and II, involved into motion by virtue of the impact interaction.

Calculate the vector $\Delta\mathbf{Q}^{(1)}$ of momentum of the element separated from the incident wave. Assume that dimensions of all the elements along the x_3 coordinate locally equal a unit. Then the element mass is $\rho^1 \cdot \xi^{(1)} \cdot \Delta t \cdot \cos\Psi^{(1)}$ and the momentum vector is $\Delta\mathbf{Q}(1) = \rho_1 \cdot \xi^{(1)} \cdot \Delta t \cdot \cos\Psi^{(1)} \cdot \dot{\mathbf{u}}^{(1)}$.

After interaction of the incident wave with the G interface the plane reflected and refracted waves are generated. In their layers of quite small thicknesses $v_i\Delta t$ behind the front planes, the field function components experienced the discontinuity (the $\mathbf{u}$ function derivatives with respect to the normal and time, as well as strains and stresses) remain constant. In the elements of these layers, formed by interaction with the element separated from the incident wave (Fig. 4), the unknown values of the media particles velocities constitute $\dot{\mathbf{u}}^{(1)}_{(1)}$, $\dot{\mathbf{u}}^{(2)}_{(1)}$ (in the reflected $qP_{(1)}$- and $qS_{(1)}$-waves) and $\dot{\mathbf{u}}^{(1)}_{(2)}$, $\dot{\mathbf{u}}^{(2)}_{(2)}$ (in the refracted $qP_{(2)}$- and $qS_{(2)}$-waves), whereas their momentum vectors are equal to

$$\begin{aligned} &\Delta\mathbf{Q}^{(1)}_{(1)} = \rho_1 \cdot \cos\Psi^{(1)}_{(1)} \cdot \xi^{(1)}_{(1)} \cdot \Delta t \cdot \dot{\mathbf{u}}^{(1)}_{(1)}, \quad \Delta\mathbf{Q}^{(2)}_{(1)} = \rho_1 \cdot \cos\Psi^{(2)}_{(1)} \cdot \xi^{(2)}_{(1)} \cdot \Delta t \cdot \dot{\mathbf{u}}^{(2)}_{(1)}, \\ &\Delta\mathbf{Q}^{(1)}_{(2)} = \rho_2 \cdot \cos\Psi^{(1)}_{(2)} \cdot \xi^{(1)}_{(2)} \cdot \Delta t \cdot \dot{\mathbf{u}}^{(1)}_{(2)}, \quad \Delta\mathbf{Q}^{(2)}_{(2)} = \rho_2 \cdot \cos\Psi^{(2)}_{(2)} \cdot \xi^{(2)}_{(2)} \cdot \Delta t \cdot \dot{\mathbf{u}}^{(2)}_{(2)}. \end{aligned} \tag{21}$$

Vector of the medium particle displacement behind the wave front is represented by (3). Taking into account this equality and correlations (21) one gains

$$\begin{aligned} &\Delta\mathbf{Q}^{(1)} = \rho_1 \cdot \cos\Psi^{(1)} \cdot \xi^{(1)} \cdot \Delta t \cdot \dot{u}^{(1)}(A^{(1)}_1\mathbf{i}_1 + A^{(1)}_2\mathbf{i}_2), \\ &\Delta\mathbf{Q}^{(r)}_{(i)} = \rho_i \cdot \cos\Psi^{(r)}_{(i)} \cdot \xi^{(r)}_{(i)} \cdot \Delta t \cdot \dot{u}^{(r)}_{(i)}(A_1(i)^{(r)}\mathbf{i}_1 + A^{(r)}_{2(i)}\mathbf{i}_2) \quad (i = 1, 2). \end{aligned} \tag{22}$$

Here $A^{(r)}_{1(i)}$, $A^{(r)}_{2(i)}$ are the projections of the corresponding vector of polarization on corresponding axes; $r = 1, 2$ identifies the wave polarization type; $i = 1, 2$ is the medium number.

Inasmuch as the momentum of the elements of media I and II, involved into motion after interaction of the incident wave with the plane G, does not change in consequence of this interaction, the condition of dynamic joining the solution in the plane G can be represented in the form

$$\Delta\mathbf{Q}^{(1)} = \Delta\mathbf{Q}^{(1)}_{(1)} + \Delta\mathbf{Q}^{(2)}_{(1)} + \Delta\mathbf{Q}^{(1)}_{(2)} + \Delta\mathbf{Q}^{(2)}_{(2)}. \tag{23}$$

It is complemented by condition (12) of compatibility of velocities of particles of media I and II in the G plane:

$$\left(\dot{\mathbf{u}} + \dot{\mathbf{u}}_{(1)}\right)\big|_G = \dot{\mathbf{u}}_{(2)}\big|_G. \tag{24}$$

Considering $\dot{u}^{(1)}$ to be prescribed, after projecting vector correlations (23), (24) on the axes Ox_1, Ox_2 one gains four scalar equations for determinations of the unknown velocities $\dot{u}^{(1)}_{(1)}$, $\dot{u}^{(2)}_{(1)}$, $\dot{u}^{(1)}_{(2)}$, $\dot{u}^{(2)}_{(2)}$. Represent them in the matrix notation:

$$A \cdot w = b_P, \tag{25}$$

where A is the matrix of dimension 4×4:

$$A = \begin{vmatrix} \rho_1\xi^{(1)}_{(1)}\cos\Psi^{(1)}_{(1)}A^{(1)}_{1(1)} & \rho_1\xi^{(2)}_{(1)}\cos\Psi^{(2)}_{(1)}A^{(2)}_{1(1)} & \rho_2\xi^{(1)}_{(2)}\cos\Psi^{(1)}_{(2)}A^{(1)}_{1(2)} & \rho_2\xi^{(2)}_{(2)}\cos\Psi^{(2)}_{(2)}A^{(2)}_{1(2)} \\ \rho_1\xi^{(1)}_{(1)}\cos\Psi^{(1)}_{(1)}A^{(1)}_{2(1)} & \rho_1\xi^{(2)}_{(1)}\cos\Psi^{(2)}_{(1)}A^{(2)}_{2(1)} & \rho_2\xi^{(1)}_{(2)}\cos\Psi^{(1)}_{(2)}A^{(1)}_{2(2)} & \rho_2\xi^{(2)}_{(2)}\cos\Psi^{(2)}_{(2)}A^{(2)}_{2(2)} \\ -A^{(1)}_{1(1)} & -A^{(2)}_{1(1)} & -A^{(1)}_{1(2)} & -A^{(2)}_{1(2)} \\ -A^{2(1)}_{2(1)} & -A^{(2)}_{2(1)} & -A^{(1)}_{2(2)} & -A^{(2)}_{2(2)} \end{vmatrix};$$

$w = \left(\dot{u}^{(1)}_{(1)} \quad \dot{u}^{(2)}_{(1)} \quad \dot{u}^{(1)}_{(2)} \quad \dot{u}^{(2)}_{(2)}\right)^{\mathrm{T}}$ is the vector of unknown values; $b_p = (\rho_1 \xi^{(1)} \cos\Psi^{(1)} A^{(1)}_1 \dot{u}^{(1)} \rho_1 \xi^{(1)} \cos\Psi^{(1)} A^{(1)}_2 \dot{u}^{(1)} A^{(1)}_1 \dot{u}^{(1)} A^{(1)}_2 \dot{u}^{(1)})^{\mathrm{T}}$ is the vector of the right-hand side, corresponding to the incident qP-wave.

To confirm the reliability of the performed transformations and the constructed correlations, consider the marginal state of the system, when a small locally plane element of the front moves along the Ox_2 axis in such a manner, that $\Psi_{(1)} = 0$ and the normal $\mathbf{n}$ is collinear to the axis Ox_2. Then the quasi-longitudinal qP-wave becomes completely longitudinal and known correlation (15) for normal interaction of a plane longitudinal shock type wave with the plane interface between isotropic elastic media should be valid. Genuinely, as in doing so $A^{(2)}_{1(1)} = -1$, $A^{(1)}_{1(1)} = A^{(1)}_{1(2)} = 0$, $\Psi^{(1)} = \Psi^{(1)}_{(1)} = \Psi^{(1)}_{(2)} = \Psi^{(2)}_{(1)} = 0$, system (25) with the matrix A disintegrates into two unjoined systems of two equations with the solution

$$\dot{u}^{(1)}_{(1)} = \frac{\rho_2 \mathrm{v}^{(1)}_{(2)} - \rho_1 \mathrm{v}^{(1)}_{(1)}}{\rho_2 \mathrm{v}^{(1)}_{(2)} + \rho_1 \mathrm{v}^{(1)}_{(1)}} \dot{u}^{(1)}, \quad \dot{u}^{(1)}_{(2)} = \frac{2\rho_1 \mathrm{v}^{(1)}_{(1)}}{\rho_2 \mathrm{v}^{(1)}_{(2)} + \rho_1 \mathrm{v}^{(1)}_{(1)}} \dot{u}^{(1)}, \quad \dot{u}^{(2)}_{(1)} = 0, \quad \dot{u}^{(2)}_{(2)} = 0, \tag{26}$$

which coincides with the solution, represented in Gol'dsmit (1960) relative to stresses.

Furthermore, the values of the medium particle velocities found with the help of (25) play the role of initial conditions and are $u^{(0)}_q(\alpha, \beta, 0)$ used in the formulae like (8) and (9).

If the incident wave is quasi-shear (qS), the technique of transformations is the same, only the expression for the momentum vector of the system before the impact changes. In this case the vector

$$\Delta\mathbf{Q}^{(2)} = \rho_1 \cdot \cos\Psi^{(2)} \cdot \xi^{(2)} \cdot \Delta t \cdot \dot{u}^{(2)} (A^{(2)}_1 \mathbf{i}_1 + A^{(2)}_2 \mathbf{i}_2)$$

is used instead of $\Delta\mathbf{Q}_{(1)}$ and Eq. (25) is replaced by

$$A \cdot w = b_S,$$

where A and w are the same but the right-side term assumes the form

$$b_S = (\rho_1 \xi^{(2)} \cos\Psi^{(2)} A^{(2)}_1 \dot{u}^{(2)} \quad \rho_1 \xi^{(2)} \cos\Psi^{(2)} A^{(2)}_2 \dot{u}^{(2)} \quad A^{(2)}_1 \dot{u}^{(2)} \quad A^{(2)}_2 \dot{u}^{(2)})^{\mathrm{T}}.$$

From the setup problem about dynamic interaction of a discontinuous wave with an interface G between anisotropic elastic media I and II, particular cases follow.

1. For example, let the plane G be a free boundary of medium I (Fig. 5). The equality $\Delta\mathbf{Q}^{(1)}_{(2)} + \Delta\mathbf{Q}^{(2)}_{(2)} = 0$ should be satisfied, according to which condition (23) is simplified to the form

   $$\Delta\mathbf{Q}^{(1)} = \Delta\mathbf{Q}^{(1)}_{(1)} + \Delta\mathbf{Q}^{(2)}_{(1)}.$$

   In this situation, only two unknown values $\dot{u}^{(1)}_{(1)}$, $\dot{u}^{(2)}_{(1)}$ remain, which are found from the system

   $$\begin{aligned} \rho_1 \xi^{(1)}_{(1)} \cos\Psi^{(1)}_{(1)} A^{(1)}_{1(1)} \dot{u}^{(1)}_{(1)} + \rho_1 \xi^{(2)}_{(1)} \cos\Psi^{(2)}_{(1)} A^{(2)}_{1(1)} \dot{u}^{(2)}_{(1)} &= \rho_1 \xi^{(1)} \cos\Psi^{(1)} A^{(1)}_1 \dot{u}^{(1)}, \\ \rho_1 \xi^{(1)}_{(1)} \cos\Psi^{(1)}_{(1)} A^{(1)}_{2(1)} \dot{u}^{(1)}_{(1)} + \rho_1 \xi^{(2)}_{(1)} \cos\Psi^{(2)}_{(1)} A^{(2)}_{2(1)} \dot{u}^{(2)}_{(1)} &= \rho_1 \xi^{(1)} \cos\Psi^{(1)} A^{(1)}_2 \dot{u}^{(1)}, \end{aligned} \tag{27}$$

   whereas Eq. (24) loses its meaning and is excluded from consideration.So it follows that when a plane qP-wave falls normally on G, system (27) has the unique solution $\dot{u}^{(1)}_{(1)} = -\dot{u}^{(1)}$, $\dot{u}^{(2)}_{(1)} = 0$. In this case, as in the theory of isotropic discontinuous waves, only one plane discontinuous $qP_{(1)}$-wave reflects, which has the same polarization and intensity, but opposite phase.
2. In the case, when elastic medium I is rigidly connected with absolutely rigid body II in the plane G, the inverse situation occurs. The equations of momentum conservation are not valid and on the strength of equalities $\dot{u}^{(1)}_{(2)} = 0$, $\dot{u}^{(2)}_{(2)} = 0$, condition (24) takes the form

   $$\left(\dot{u} + \dot{u}_{(1)}\right)\big|_G = 0.$$

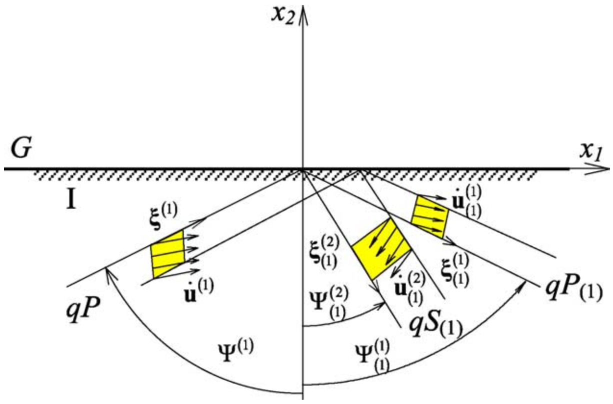

Fig. 5. Interaction of a discontinuous wave with a free boundary.

The two unknown values $\dot{u}^{(1)}_{(1)}$, $\dot{u}^{(2)}_{(1)}$ are determined through the use of the equations

$$\begin{aligned} A^{(1)}_{1(1)}\dot{u}^{(1)}_{(1)} + A^{(2)}_{1(1)}\dot{u}^{(2)}_{(1)} &= -A^{(1)}_{1}\dot{u}^{(1)}, \\ A^{(1)}_{2(1)}\dot{u}^{(1)}_{(1)} + A^{(2)}_{2(1)}\dot{u}^{(2)}_{(1)} &= -A^{(1)}_{2}\dot{u}^{(1)}. \end{aligned} \tag{28}$$

If the incident wave front is parallel to the G plane, also only one plane discontinuous qP_1-wave reflects, for which, as in the theory of isotropic discontinuous waves, the discontinuity value, polarization and phase coincide with the appropriate characteristics of the initial incident wave.

3. Now consider the case, when medium II is isotropic and is characterized by the elasticity parameters λ_2, μ_2, ρ_2. In this case all the directions are equivalent in this medium and only two kinds of waves can propagate in it. They are the purely longitudinal P-wave with the velocity $\alpha = \sqrt{(\lambda_2 + 2\mu_2)/\rho_2}$ and the purely shear S-wave with the velocity $\beta = \sqrt{\mu_2/\rho_2}$. As it takes place, the wave front surface is orthogonal to rays, the vector of ray velocity coincides with the vector of phase velocity and the velocities of particles displacements are in parallel with the rays for the P-waves or orthogonal to them for the S-waves. Calculate momenta of elements of the corresponding particles of the refracted wave, which relate to the unit segment AB of the interface G (Fig. 4)

$$\begin{aligned} \Delta\mathbf{Q}^{(1)}_{(2)} + \Delta\mathbf{Q}^{(2)}_{(2)} = {} & \alpha \cdot \Delta t \cdot \rho_2 \cdot \dot{u}^{(1)}_{(2)} \cdot \cos\Theta^{(1)}_{(2)}(\sin\Theta^{(1)}_{(2)} \cdot \mathbf{i}_1 - \cos\Theta^{(1)}_{(2)} \cdot \mathbf{i}_2) + \beta \cdot \Delta t \cdot \rho_2 \cdot \dot{u}^{(2)}_{(2)} \\ & \cdot \cos\Theta^{(2)}_{(2)}\left(-\cos\Theta^{(2)}_{(2)} \cdot \mathbf{i}_1 - \sin\Theta^{(2)}_{(2)} \cdot \mathbf{i}_2\right). \end{aligned} \tag{29}$$

With allowance made for this equality the A matrix is transduced as follows:

$$\left\| \begin{matrix} \rho_1\xi^{(1)}_{(1)}\cos\Psi^{(1)}_{(1)}A^{(1)}_{1(1)} & \rho_1\xi^{(2)}_{(1)}\cos\Psi^{(2)}_{(1)}A^{(2)}_{1(1)} & \alpha\rho_2\cos\Theta^{(1)}_{(2)}\sin\Theta^{(1)}_{(2)} & -\beta\rho_2\cos^2\Theta^{(2)}_{(2)} \\ \rho_1\xi^{(1)}_{(1)}\cos\Psi^{(1)}_{(1)}A^{(1)}_{2(1)} & \rho_1\xi^{(2)}_{(1)}\cos\Psi^{(2)}_{(1)}A^{(2)}_{2(1)} & -\alpha\rho_2\cos^2\Theta^{(1)}_{(2)} & -\beta\rho_2\cos\Theta^{(2)}_{(2)}\sin\Theta^{(2)}_{(2)} \\ -A^{(1)}_{1(1)} & -A^{(2)}_{1(1)} & \sin\Theta^{(1)}_{(2)} & -\cos\Theta^{(2)}_{(2)} \\ -A^{(1)}_{2(1)} & -A^{(2)}_{2(1)} & \cos\Theta^{(1)}_{(2)} & \sin\Theta^{(2)}_{(2)} \end{matrix} \right\|.$$

The angles $\Theta^{(1)}_{(2)}$, $\Theta^{(2)}_{(2)}$ are easily determined via the Snell equations

$$\frac{\sin\Theta^{(1)}}{\mathrm{v}^{(1)}} = \frac{\sin\Theta^{(1)}_{(2)}}{\alpha} = \frac{\sin\Theta^{(2)}_{(2)}}{\beta}.$$

Inasmuch as the velocities α and β are known beforehand, one has

$$\Theta^{(1)}_{(2)} = \arcsin\left(\frac{\alpha \cdot \sin\Theta^{(1)}}{\mathrm{v}^{(1)}}\right), \quad \Theta^{(2)}_{(2)} = \arcsin\left(\frac{\beta \cdot \sin\Theta^{(1)}}{\mathrm{v}^{(1)}}\right).$$

4. Let the medium II be ideal compressible liquid (Fig. 6). Inasmuch as it cannot perceive shear stresses, the $S_{(2)}$-wave is absent ($\dot{u}^{(2)}_{(2)} = 0$) and cohesion between the liquid and the elastic medium in the direction of the Ox_1 axis is lost. In this case the displacements and velocities of particles of the liquid and the solid, directed along the Ox_1 axis, do not coincide typically in the separating plane G. For this reason the third equation of system (25) loses its validity and in matrix A the third line and the fourth column disappear, so it takes the form:

$$A = \left\| \begin{matrix} \rho_1\xi^{(1)}_{(1)}\cos\Psi^{(1)}_{(1)}A^{(1)}_{1(1)} & \rho_1\xi^{(2)}_{(1)}\cos\Psi^{(2)}_{(1)}A^{(2)}_{1(1)} & \alpha\rho_2\cos\Theta^{(1)}_{(2)}\sin\Theta^{(1)}_{(2)} \\ \rho_1\xi^{(1)}_{(1)}\cos\Psi^{(1)}_{(1)}A^{(1)}_{2(1)} & \rho_1\xi^{(2)}_{(1)}\cos\Psi^{(2)}_{(1)}A^{(2)}_{2(1)} & -\alpha\rho_2\cos^2\Theta^{(1)}_{(2)} \\ -A^{(1)}_{2(1)} & -A^{(2)}_{2(1)} & \cos\Theta^{(1)}_{(2)} \end{matrix} \right\|. \tag{30}$$

The unknown vector w and the vector b_{p} of the right member of the equation system are reduced to the form:

$$w = \left(\dot{u}^{(1)}_{(1)} \quad \dot{u}^{(2)}_{(1)} \quad \dot{u}^{(1)}_{(2)}\right)^{\mathrm{T}},$$
$$b_p = \left(\rho_1\xi^{(1)}\cos\Psi^{(1)}A^{(1)}_1\dot{u}^{(1)} \quad \rho_1\xi^{(1)}\cos\Psi^{(1)}A^{(1)}_2\dot{u}^{(1)} \quad A^{(1)}_2\dot{u}^{(1)}\right)^{\mathrm{T}}.$$

So, solving vector equation (25) with coefficient matrix (30), it is possible to find the three components of the required vector w.

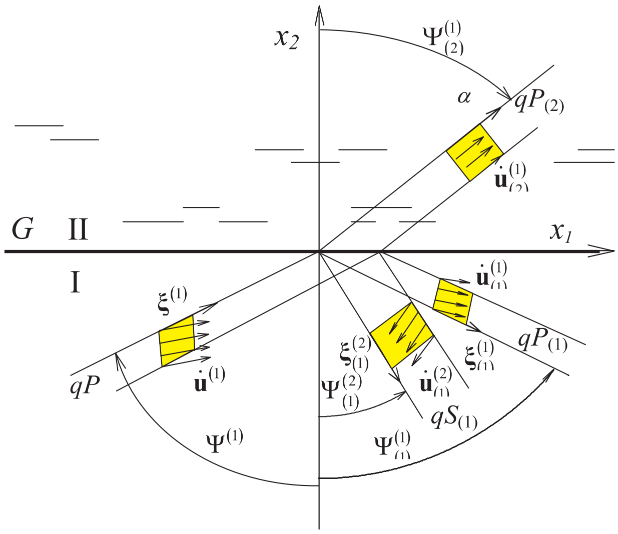

Fig. 6. Interaction of a discontinuous wave with an interface between anisotropic elastic and liquid media.

5. Results of analysis

With the aid of the proposed technique, the problem of propagation and dynamic interaction of shock-type waves with plane and curvilinear interfaces G between transversally isotropic media is studied.

In the first case the source of the shock wave in medium I is a spherical cavity C with the unit radius and its center $x_1 = x_2 = x_3 = 0$ located at a distance 10 m from the interface plane G. The plane is perpendicular to the Ox_2 axis, which coincides with the axis of symmetry of the elasticity parameters of the media I and II. Owing to the symmetry properties, the components $c_{ik,pq}$ of the tensor of the elasticity constants are conveniently represented as a six-row square matrix $(C_{\alpha\beta})$. The correspondence of their elements is established by the scheme

$$(11) \leftrightarrow 1; \quad (22) \leftrightarrow 2; \quad (33) \leftrightarrow 3; \quad (23) = (32) \leftrightarrow 4; \quad (13) = (31) \leftrightarrow 5; \quad (12) = (21) \leftrightarrow 6.$$

Inasmuch as the elastic properties of a transversely isotropic medium are characterized by five irreducible parameters, the $(C_{\alpha\beta})$ matrix is represented as

$$(C_{\alpha\beta}) = \begin{pmatrix} \lambda + 2\mu & \lambda & \lambda - l & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda - l & 0 & 0 & 0 \\ \lambda - l & \lambda - l & \lambda + 2\mu - p & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu - m & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu - m & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix}. \tag{31}$$

It follows from (31) that the deviation of the examined media from isotropic ones is determined by the three parameters l, m, p, where λ and μ are the Lame parameters.

It is assumed that a normal pressure with intensity $p_0 = 10^9$ Pa is instantaneously applied to the cavity surface, which generates not only a quasi-longitudinal (qP), as in isotropic media, but also quasi-shear (qS) shock-type waves with axially symmetrical front surfaces. Owing to all this, the intensity of the quasi-shear wave, which is polarized orthogonally to the first two, is equal to zero. The front surface of the other qS-wave lags behind the qP-wave front and it is not discussed here.

After interaction with the G interface, also only two types of each of axisymmetric reflected and refracted waves polarized in the plane of axial cross-section are initiated.

The materials of the media have the elastic characteristics $\lambda_1 = 4.972 \times 10^{10}$ Pa, $\mu_1 = 3.906 \times 10^{10}$ Pa, $\rho_1 = 2650$ kg/m^3 and $\lambda_2 = 3.409 \times 10^9$ Pa, $\mu_2 = 1.364 \times 10^{10}$ Pa, $\rho_2 = 2760$ kg/m^3. The quantities l, m, p, which disturb the isotropy properties are chosen to be $l_1 = -0.5\lambda_1$, $m_1 = -0.4\mu_1$, $p_1 = -0.5(\lambda_1 + 2\mu_1)$, $l_2 = 0.5\lambda_2$, $m_2 = 0.3\mu_2$, $p_2 = 0.1(\lambda_2 + 2\mu_2)$.

In Fig. 7, the lines of fronts of the incident ($qP - 1$), reflected ($qP_{(1)} - 2$ and $qS_{(1)} - 3$) and refracted ($qP_{(2)} - 4$ and $qS_{(2)} - 5$) waves are shown for the selected time instant. The values of the media particles velocities $\dot{u}^{(1)}$, $\dot{u}^{(1)}_{(1)}$, $\dot{u}^{(2)}_{(1)}$, $\dot{u}^{(1)}_{(2)}$, $\dot{u}^{(2)}_{(2)}$ are plotted on the corresponding lines in the same scale. It can be seen, that the intensity of the refracted $qP_{(2)}$-wave exceeds the intensities of the qP- and $qP_{(1)}$-wave at the Ox_2 axis. This effect does not contravene equations (26) with additional comment, that the qP- and $qP_{(1)}$-wave have moved to greater distances from the G plane and have lost the greater parts of their intensities.

Solving the problem of twofold interaction of a discontinuous wave with curvilinear interfaces permits one to simulate the phenomena of focusing and dissipation of a plane wave front by elastic anisotropic hyperboloid lenses. Then for equations (11) and (25) to be used, they are formulated in coordinate systems locally slewed through appropriate angles. In Fig. 8, the results of the construction of a shock wave front transformation by an elastic lens are demonstrated. The plane discontinuous P-wave front is propagating along the Ox_2 axis in medium I with the property parameters $\lambda_1 = 4.972 \times 10^{10}$ Pa, $\mu_1 = 3.906 \times 10^{10}$ Pa,

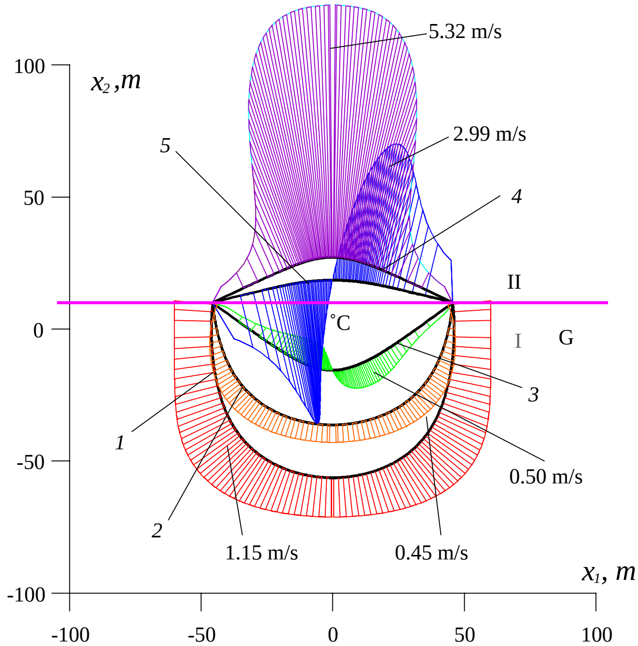

Fig. 7. Diffraction of a discontinuous wave at the plane interface.

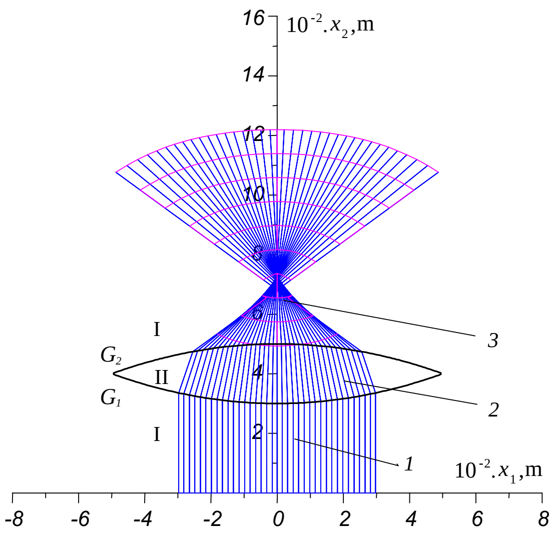

Fig. 8. Focusing a discontinuous wave by elastic lens.

$l_1 = -0.6\lambda_1$, $m_1 = 0.2\ \mu_1$, $p_1 = 0.1(\lambda_1 + 2\mu_1)$, $\rho_1 = 2650\ \text{kg/m}^3$. Then it interacts with the hyperboloid interface G_1 between the initial medium and medium II (hyperboloidal lens) with the parameters $\lambda_2 = 3.977 \times 10^9$ Pa, $\mu_2 = 1.591 \times 10^{10}$ Pa, $l_2 = -0.3\lambda_2$, $m_2 = -0.2\mu_2$, $p_2 = 0.1(\lambda_2 + 2\mu_2)$, $\rho_2 = 2760\ \text{kg/m}^3$. In consequence of this, the wave rays deflect their directions and the wave front becomes curvilinear. Thereupon

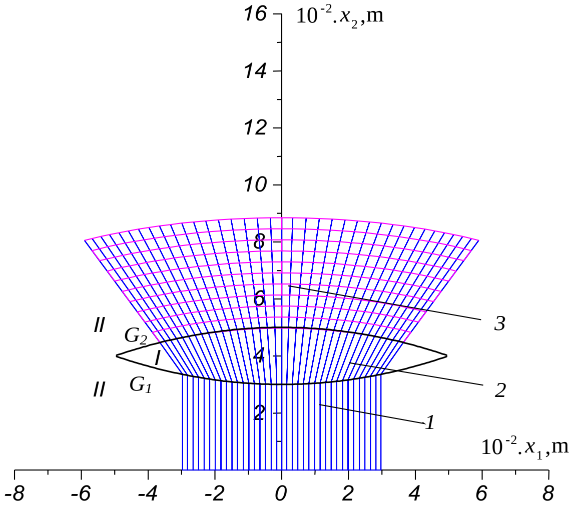

Fig. 9. Scattering a discontinuous wave by elastic lens.

after interaction of the transformed wave front with the second interface surface G_2, the phenomenon of the wave front focusing is completed. Note, that the problem about dynamic interaction of a discontinuous wave with two curvilinear interfaces presents complicated calculational difficulties, inasmuch as four waves generated supplementally on the first interface and eight waves on the second one should be calculated in addition. Therefore the problem of tracing all of them is rather cumbersome. Because of this, in Fig. 8 the fronts of the first qP-wave are plotted only, as the fronts of other waves lag behind the selected qP-wave fronts and do not affect them.

Besides, it is impossible to calculate the wave intensity at the focal points, because the ray divergence J tends to zero (see (9)) at them and the procedure of division in equality (9) loses its sense. For this reason the $u_q^{(0)}$ value increases endlessly (in the frame work of the theory of ideal elasticity) and the wave intensity acquires infinite value.

But if to change media I and II by their places (Fig. 9), the discontinuous wave is scattered by the lens, the J value enlarges and according to (9) the wave intensity $u_q^{(0)}$ decreases.

6. Conclusions

The problem about discontinuous wave interaction with the interface between anisotropic elastic media, accompanied by formation of reflected and refracted quasi-longitudinal and quasi-transverse discontinuous waves, is considered on the basis of the ray method of geometrical optics. It is associated with the questions of geometrical construction of evolving fronts of moving field function discontinuities and calculation of their magnitudes presenting the most comprehensive information about intensity of the impulse carried by the wave at every point of the front. The answer to the first question is provided with the use of the Snell equations. To give the answer to the second one, the zeroth approximation of the ray method is used.

The approach outlined is especially effective in the vicinity of interfaces between anisotropic elastic media because it permits one to use general correlations of momentum conservation principle of the theory of stereomechanical impact, formulated for the media particles involved into motion owing to impact interaction of the incident, reflected and refracted waves. With such a method, the constitutive equations of the incident discontinuous wave interaction with a free surface, absolutely rigid body, liquid and other elastic isotropic or anisotropic media are formulated. Software is elaborated for computer simulation of the considered processes.

With the aid of the proposed technique, the phenomenon of evolution of a discontinuous wave generated by a shock impulse in a spherical cavity in a transversely isotropic medium is traced. Geometrical construction of the wave fronts and the calculation of the wave discontinuity values are performed for different time instants. Dynamic interaction of the wave with a plane interface is investigated. It is established that the intensity of the refracted qP-wave exceeds the intensity of the incident qP-wave for the chosen values of the media parameters.

The effects of discontinuous waves penetrations through elastic anisotropic lenses are analysed. It is shown, that depending on the elastic properties of the lenses and ambient medium, they can focus or scatter the discontinuous waves. As this takes place, dynamic stresses in the focal points tend to infinity in the framework of the theory of ideal elasticity.

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